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Structural Synthesis—Its Genesis and Development

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Introduction

THE need to reduce structural weight without compromising structural integrity, particularly in aerospace applications, has historically been a strong driving force behind the development of optimum design methods. Today, the need for energy conservation in transportation systems, via weight reduction, is perhaps an even more compelling motivation for application of structural optimization methods. Furthermore, the growing use of fiber composite materials in structures is likely to increase demand for modern analytical tools that will make it possible to fully exploit the design potential offered by these new materials.

Huge strides have been made in the development of reliable structural analysis methods during the past 30 years. The steady and continuing growth of digital computing power, accompanied by ever lower cost per operation, has facilitated the development of rather general structural analysis capabilities such as the various finite element programs (e.g., NASTRAN, ASKA, SPAR). As confidence has grown in our ability to predict the behavior of alternative designs, there has been a natural tendency to face the challenging problems of wider scope associated with the structural design process. It will be assumed herein that we know how to predict the behavior of some structural systems well enough to undertake structural optimization.

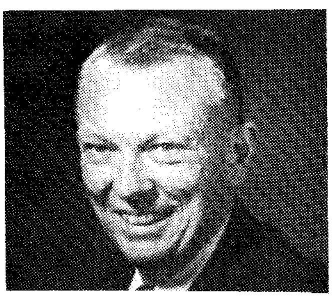
In what follows, my objective will be to offer a rather personal perspective of how structural optimization has changed and matured over the last two decades. This task will be attacked by first introducing some basic definitions and fundamental concepts. The state-of-the-art prior to 1958 will be reviewed, and then I will recall some of the early experiences which had an influence on the thinking that led me to set forth the structural synthesis concept in 1960. Subsequently, I will trace my role and recall the influence of other scholars on the growth and development of structural synthesis since 1960. A particularly exciting aspect of this period involves the productive competition between two schools of thought in structural optimization which raged during the 70's.

Basic Definitions and Fundamental Concepts*

It will be useful to agree upon some basic definitions in order to facilitate an understanding of fundamental structural optimization concepts. An idealized structural system can be described by a finite set of quantities that specify the materials, the arrangement and the dimensions of the structure. *Preassigned parameters* are those quantities describing a structural system that are fixed at the outset. They are not to be varied by the redesign procedure. *Design variables* are those quantities defining a structural system that are varied by the design modification procedure. The term *load condition* refers to one of several distinct sets of mechanical and thermal loads that approximately represent the effect on the structure of the environment to which it is exposed. A *failure mode* is defined as any structural behavior characteristic subject to limitation by the responsible engineer. A rather broad class of failure modes which includes limitations on stress, deflection, buckling, natural frequency, dynamic instability and other behavioral characteristics can be formulated as inequality constraints. An *objective function* is defined as a function of the design variables which provides a basis for choice between alternative acceptable designs. While weight minimization is often taken as the objective in structural optimization, it should be noted that general methods of structural optimization do not inherently require the use of weight as an objective function.

The basic structural synthesis concept is now introduced using elementary examples involving only two independent design variables. These simple examples are used to help fix ideas while initially avoiding mathematical abstraction and the associated generality.

The first example is a simple *component design* optimization problem. Consider the thin-walled simply supported tubular column shown schematically in Fig. 1. The column is subjected to a single *load condition* represented by $P = 5000$ lb. The length ($\ell = 100$ in.), modulus of elasticity ($E = 10 \times 10^6$ lb/in.^{2}), and density ($\rho = 0.1$ lb/in.^{3}) are *preassigned* parameters. The mean diameter $[D = (D_0 + D_i)/2]$ and the wall thickness T are taken to be the two independent



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*This section is intended to provide a common point of departure for a broadly based readership. Individual readers active in the structural optimization field may wish to skip ahead to the next section.

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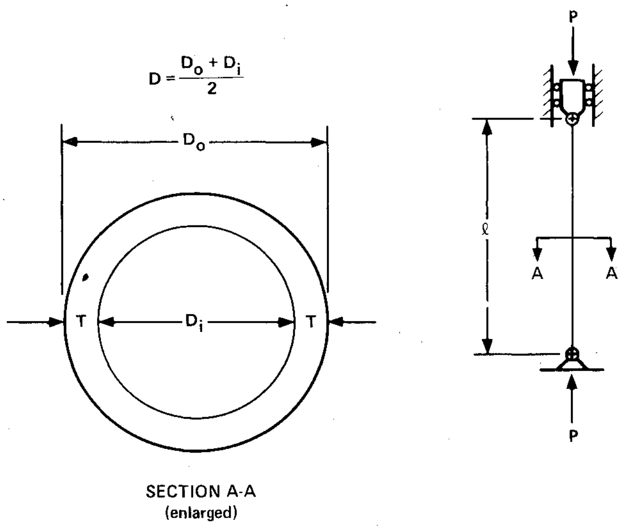

Fig. 1 Simple component design example—thin-walled tubular column.

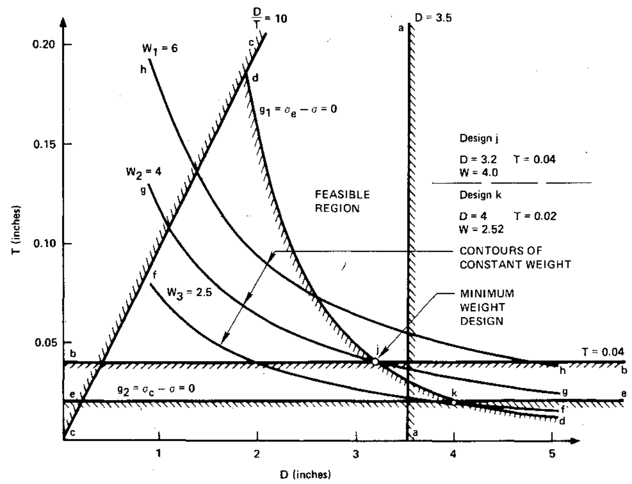

Fig. 2 Design space map for tubular column example.

design variables. The problem posed is to find D and T such that the weight of the column

$$W = \rho\pi\ell DT \rightarrow \text{Min} \tag{1}$$

while requiring that the following *side constraints*:

$$D \leq D_{max} = 3.5 \text{ in.} \tag{2}$$

$$T \geq T_{min} = 0.04 \text{ in.} \tag{3}$$

$$(D/T) \geq 10 \tag{4}$$

and *behavior constraints*:

$$g_1(D,T) = \sigma_e - \sigma = \frac{\pi^2 E}{8\ell^2}(D^2 + T^2) - \frac{P}{\pi DT} \geq 0 \tag{5}$$

$$g_2(D,T) = \sigma_c - \sigma = \frac{0.4ET}{D} - \frac{P}{\pi DT} \geq 0 \tag{6}$$

be satisfied. Note that the behavior constraints embodied in Eqs. (5) and (6) guard against the Euler and local buckling (crippling) *failure modes*, respectively; whereas the side constraints in Eqs. (2-4) impose practical geometric requirements on the design.

The region of all possible positive values of D and T shown in Fig. 2 is referred to as a two-dimensional design space. The side constraints can now be given graphical interpretation as follows: all design points to the left of line a-a, above line b-b, and below as well as to the right of line c-c satisfy the constraints embodied in Eqs. (2), (3), and (4), respectively. The set of all designs satisfying the side constraints is further reduced by also imposing the behavior constraints. That is, points in the design space above and to the right of line d-d satisfy the Euler buckling constraint [Eq. (5)] and design points above the line e-e satisfy the local buckling (crippling) constraint [Eq. (6)]. Note that for the example at hand (with $T_{min} = 0.04$ in.) the local crippling constraint is redundant, since the side constraint imposed by Eq. (3) is more restrictive. The set of points that satisfies all of the side and behavior constraints is referred to as the *feasible region* of the design space. In Fig. 2 the curved lines h-h, g-g, and f-f represent three distinct contours of constant weight corresponding, respectively, to $W_1 = 6$ lb, $W_2 = 4$ lb, and $W_3 = 2.5$ lb.

Since the objective function and the constraints are all *explicit functions* of the two independent *design variables* (D and T), plotting the constraint boundaries and the objective contours in the design space (as shown in Fig. 2) is straightforward. The optimum design is easily found by scanning the design space map (Fig. 2) with the mind's eye, seeking out the minimum weight design point in the feasible region, namely the point j in Fig. 2 ($D = 3.2$ in.; $T = 0.04$ in.; $W = 4.0$ lb). In this instance, the minimum weight optimum design happens to reside at the vertex formed by the intersection of the Euler buckling constraint $[g_1(D,T) = 0]$ and the lower limit on the tube wall thickness ($T = 0.04$ in.). However, if the side constraint limits are changed so that $D_{max} = 5$ in. and $T_{min} = 0.015$ in., then the minimum weight optimum design is seen to be point k in Fig. 2 ($D = 4$ in., $T = 0.02$ in., $W = 2.52$ lb). In this case, the minimum weight optimum design happens to reside at the vertex formed by the intersection of the Euler buckling constraint $[g_1(D,T) = 0]$ and the local buckling (crippling) constraint $[g_2(D,T) = 0]$. The important point made by this example is that when a structural optimization problem is posed initially, it is not generally known which inequality constraints will become critical (equality) constraints at the optimum design.

The foregoing results might lead us to believe that, although we can not tell in advance which constraints are going to be critical, the optimum design always corresponds to a vertex point in the design space. This, however, is not a valid surmise, because in general the behavior constraint functions and the objective function are nonlinear. This point is illustrated clearly by our second example, a simple *system level design* optimization problem. Consider the symmetric two-member planar truss shown in Fig. 3 subject to a single *load condition* represented by $2P = 66{,}000$ lb. Let the two identical tubular members have annular cross sections with preassigned wall thickness $T = 0.1$ in. The horizontal distance

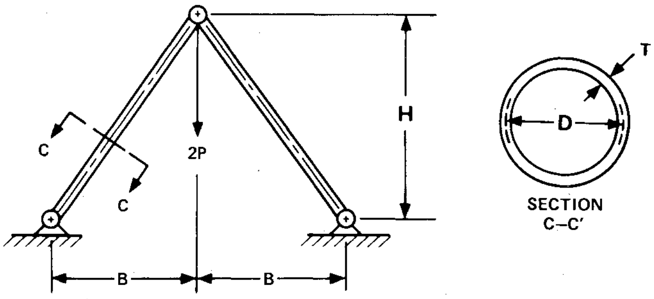

Fig. 3 Simple system level design example—symmetric two-member truss.

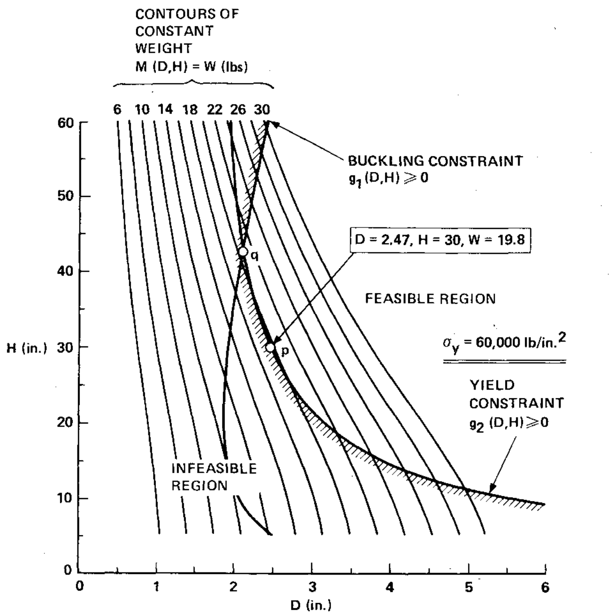

Fig. 4 Design space map for two-member truss (σ_y = 60,000 lb/in.^{2}).

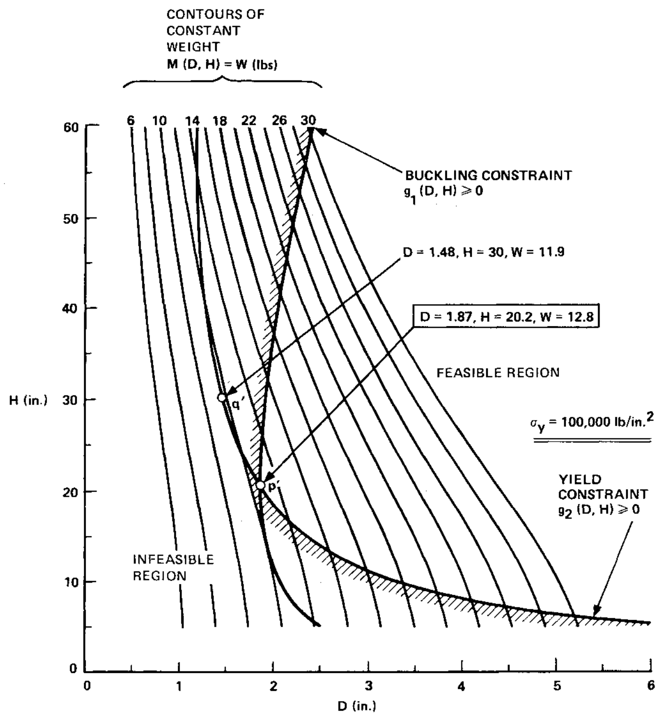

Fig. 5 Design space map for two-member truss (σ_y = 100,000 lb/in.^{2}).

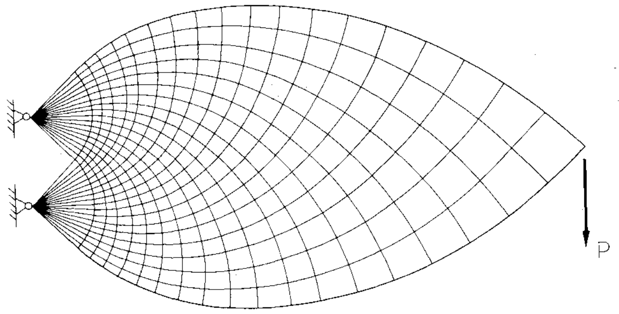

Fig. 6 Representative Michell-type structural layout.[4]

between the support points ($2B = 60$ in.) and the pertinent material properties are also *preassigned parameters* as follows: modulus of elasticity $E = 30 \times 10^6$ lb/in.^{2}, density $\rho = 0.3$ lb/in.^{3}, and yield stress $\sigma_y = 60{,}000$ lb/in.^{2} The mean diameter of the tubular members D and the height H of the truss are taken to be the two independent *design variables*. The problem posed is to find D and H such that the weight of the truss system

$$W = 2\rho\pi DT(B^2 + H^2)^{1/2} \rightarrow \text{Min} \qquad (7)$$

while requiring that the following behavior constraints be satisfied

$$g_1(D,H) = \sigma_e - \sigma = \frac{\pi^2 E(D^2 + T^2)}{8(B^2 + H^2)} - \frac{P(B^2 + H^2)^{1/2}}{\pi TDH} \geq 0 \qquad (8)$$

$$g_2(D,H) = \sigma_y - \sigma = \sigma_y - \frac{P(B^2 + H^2)^{1/2}}{\pi TDH} \geq 0 \qquad (9)$$

Note that the behavior constraints embodied in Eqs. (8) and (9) guard against Euler buckling and yield condition *failure modes*, respectively. In this example, no side constraints are specified.

The region of all possible positive values of D and H shown in Fig. 4 represents the two-dimensional design space for the two-bar-truss system example. By setting $g_1(D,H) = 0$ and $g_2(D,H) = 0$ [see Eqs. (8) and (9)], we can plot the critical behavior constraint curves in the design space and observe that they divide it into a feasible and infeasible region. Also, by setting W in Eq. (7) equal to various specific values (i.e., 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30 lb), we can construct contours of constant weight in the design space, as shown in Fig. 4. Scanning the design space map in Fig. 4, we see that the minimum weight design is at point p where $D = 2.47$ in., $H = 30$ in., and $W = 19.8$ lb. In this instance the optimum design is not at a vertex; rather it is seen that the only critical constraint at point p is the yield stress limitation (i.e., at the design point p the stress $\sigma = \sigma_y = 60{,}000$ lb/in.^{2}). It is interesting and important to observe that if the yield stress limit is raised to $\sigma_y = 100{,}000$ lb/in.^{2} (say by heat treating the material), while the rest of the optimum design problem statement remains unchanged, then the design space map is modified to that shown in Fig. 5, where it is now apparent that the minimum weight optimum design is at the point p ($D = 1.87$ in., $H = 20.2$ in., and $W = 12.8$ lb). Thus, in this case, the optimum design happens to reside at the vertex formed by the intersection of the Euler buckling and the yield stress behavior constraints.

These two simple examples clearly reveal that, *in general, it cannot be anticipated how many or which constraints will be critical at the optimum design.* Because of this the use of inequality constraints is essential to the proper statement of the general structural design optimization problem.

State-of-the-Art Prior to 1958

The structural optimization literature as of 1958 can be usefully divided into three main categories as follows: 1) the classical literature dealing with the least weight layout of highly idealized frameworks; 2) optimum design of structural *components* based on weight-strength analysis or structural index methods; and 3) minimum weight optimum design of simple structural *systems* based on the plastic collapse or limit analysis design philosophy.

The early work of Maxwell,[1] and the subsequent development by Michell,[2] provided a basic theory for the optimal layout of minimum weight trusses under a single load condition and subject to stress constraints only. Michell's work provided a basis for generating an optimal orthogonal layout given the positions, in a two-dimensional space, of the load application and support points. A representative Michell-type layout is shown in Fig. 6. The resulting structures are statically determinate and potentially unstable if alternative loads are applied. Michell structures usually involve an indefinitely large number of infinitesimal members, so that they are seldom suitable for direct use in engineering design. However, Michell structures can provide useful guidance for the layout of structural systems, particularly when a single load condition is dominant and stress constraints are of primary concern. The classical layouts can also provide a reference solution for assessing the efficiency of practical configurations. During the late 1950's the further development of Michell-type least-weight structures was pursued by Hemp[3] and Chan.[4] The design of Michell structures is analogous to the analysis of the slip line fields associated with the flow of rigid-perfectly-plastic materials. In Ref. 4 Chan employed a graphical technique developed for the analysis of slip line fields to obtain Michell-type least weight designs.

Minimum weight optimum design of basic aircraft structural components, such as columns and stiffened panels, subject to compressive loads was initially developed during World War II. Two of the earliest contributions were those of Cox and Smith[5] and Zahorski.[6] Subsequently, during the late 1940's and the early 1950's a great deal of effort went into the development of minimum weight design methods for aircraft structural components subject to buckling constraints. By 1958 both Shanley[7] and Gerard[8] had published books dealing primarily with the minimum weight optimum design of aircraft structural components. An excellent chronological bibliography covering the period from 1940-1956 is contained in an Appendix of Ref. 8.

The basic approach followed in this early optimization work on aircraft structural components can be characterized as the "simultaneous failure mode design optimization method," wherein a structural component is proportioned so that several preselected failure modes become critical simultaneously. Setting the number of simultaneously critical failure modes equal to the number of independent design variables *converted* the design optimization problem from an inequality constrained weight minimization problem to a set of *nonlinear simultaneous equations.* From a design space point of view, the solutions obtained by the simultaneous failure mode approach correspond to a *preselected vertex point.* See, for example, design point k in Fig. 2, assuming the side constraints have been relaxed so that $T_{\min} = 0.015$ in. and $D_{\max} = 5$ in. By and large, the success of these methods depended on: 1) sound intuition and good physical insight for an inspired choice of the "correct set" of critical constraints, and 2) the fact that even when the true optimum was not at a vertex the error in the weight was often small. This second point is illustrated by the vertex point q in Fig. 4 which is not the optimum design, but has a weight only 5% higher than that of the true optimum at point p.

For aircraft structural components such as columns, wide columns, and stiffened panels subject to a single loading condition, the simultaneous failure mode approach led to a set of constraint *equations* that could often be solved explicitly for the design variable values at the preselected vertex point. These values frequently corresponded to the minimum weight optimum design, and they were commonly expressed as functions of some appropriate measure of loading intensity (e.g., P/L^2 for a column, P/LB for a wide column, and N_x/L for a panel), which was referred to as a "structural or loading index" (Ref. 7). The significance of the "structural index" idea resides in the fact that it is selected to be a measure of loading intensity for which stress distributions will be identical in all geometrically similar elements. Thus, in addition to providing explicit and frequently exact optimum design solutions, the simultaneous failure mode approach provided results that were applicable to an entire class of components. The "structural index" concept provided a valuable tool for comparing the weight efficiency of alternative materials and design configurations for structural components as illustrated by Fig. 7 (drawn from Ref. 7, p. 34). It is important to recognize that the structural index concept is independent of the method used to obtain the data. Therefore, results generated by the simultaneous failure mode approach, experimental measurement, as well as modern structural synthesis methods can all be presented in summary form via plots of mass index vs loading index.

As of 1958, the "structural index" approach to aircraft structural optimization was a highly developed and useful design tool; however, the following shortcomings were evident:

1) The method was limited to relatively simple components.

2) It was limited to consideration of a single simple loading condition (e.g., uniform compression or shear).

3) Constraints other than stress and buckling constraints (e.g., side constraints and stiffness type constraints) were not treated directly.

4) The method was based on the simultaneous failure mode approach, which was intuitive and useful but potentially flawed when viewed from the design space point of view.

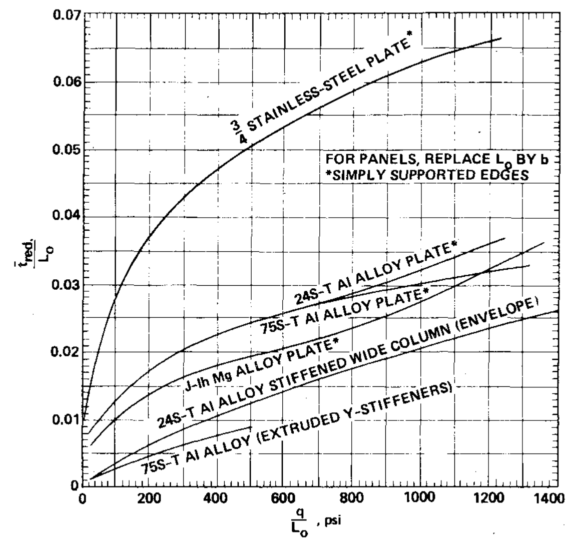

Fig. 7 Weight index vs load index for wide columns and panels.[7]

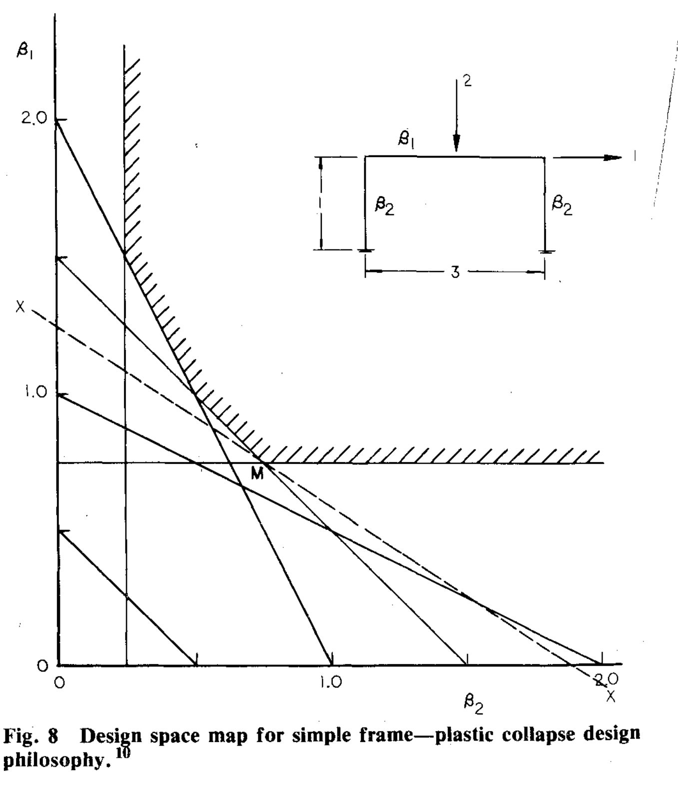

Fig. 8 Design space map for simple frame—plastic collapse design philosophy.[10]

Prior to 1958, the application of mathematical programming algorithms to structural optimization was limited to truss and planar frame type problems that could be formulated within the context of the plastic collapse design philosophy. Briefly stated, this design philosophy seeks to minimize the weight while precluding plastic collapse of the structure when it is subjected to one or more overload conditions obtained by scaling up service load conditions. Within the context of the plastic collapse design philosophy a significant class of structural optimization problems could be formulated as linear programming problems. The early applications of linear programming to the minimum weight design of planar frames, based on the plastic collapse design philosophy, were made by Heyman,[9] Foulkes,[10] Prager,[11] and Livesley.[12] These investigations considered only a single load condition. Figure 8, from Ref. 10, may be interpreted as a two-dimensional design space for minimum weight design of the simple frame shown, based on the plastic collapse design philosophy. In the notation of Ref. 10, β_1 and β_2 denote the fully plastic moments for the girder and column members, respectively, and the weight of the frame is given by $G=3\beta_1+2\beta_2$. Each of the six solid straight lines in Fig. 8 represents a linear inequality constraint associated with precluding a distinct collapse mode, and the dashed line is the weight contour which passes through the minimum weight optimum design point $M[\beta_1=\beta_2=3/4,\ G=15/4]$. Design points above and to the right of the cross-hatched lines in Fig. 8 represent feasible designs that will not collapse. (N.B.: the cross hatching is inside the feasible region.) The fact that these linear programming problems had optimum design solutions corresponding to vertex points in the design space was probably reassuring to users of the simultaneous failure mode approach.

Subsequently, the need for including multiple or alternative loading conditions was recognized and dealt with successfully by Heyman and Prager[13] as well as Livesley.[14] These early applications of mathematical programming methods to optimum structural design took the form of linear programming problems because they were formulated within the simplifying framework of the plastic collapse design philosophy. This type of system level structural optimization, generated in a civil engineering context, was focused primarily on steel frame structures and it did not consider stress, deflection, or buckling constraints under service load conditions.

In retrospect, there are two papers that deserve special attention here. As early as 1955, Klein[15] had recognized that a more general class of structural design optimization problems could be viewed as nonlinear mathematical programming problems. Although Ref. 15 did not consider multiple load conditions, the fundamental importance of *inequality constraints* in properly stating structural design optimization problems was recognized. The influence of Ref. 15 was probably limited by the fact that Klein solved his example problems by using classical Lagrange multiplier and slack variable concepts to transform the inequality constrained weight minimization problem into a set of nonlinear simultaneous equations. The resulting large number of equations and unknowns, as well as the apparent need to exhaustively sort through all of the solutions, was a grim prospect when larger, more realistic design problems were contemplated. The severe criticism by the operations research community (see Refs. 15A, B, C) of Klein's recourse to classical solution techniques (rather than to nonlinear programming algorithms) probably discouraged him from continuing his studies in the optimum structural design area.

In 1958, Pearson working within the plastic collapse design philosophy, treated the minimum weight design of truss and frame structures subject to a multiplicity of distinct overload loading conditions.[16] Although this work ignores stress, deflection, and buckling constraints under service load conditions, it is important because the innovative solution method employed was a precursor of:

1) The integrated approach to structural analysis/synthesis, where these two activities are carried out simultaneously rather than sequentially.

2) The conversion of an inequality constrained optimization problem to one or more equivalent unconstrained minimization problems.

3) Reducing the dimensionality of the space in which the bulk of the numerical calculations are to be made via imaginative changes of variable.

The essential features of Pearson's approach can be illustrated by considering the plastic collapse design of a general truss structure formulated as follows:

Let A_i denote the cross-sectional area of the ith truss member, F_{ij} the force in the ith truss member under the jth load condition, and R_{kj} the value of the kth redundant force under the jth load condition. Given the yield stresses in tension, σ_i^+ and compression σ_i^- for each member i, as well as the geometric configuration and the load conditions, find the R_{kj} such that

$$\sigma_i^- A_i \le F_{ij} \le \sigma_i^+ A_i \tag{10}$$

$$W(R_{kj}) = \sum_{i=1}^{I} \rho_i L_i A_i \rightarrow \text{Min} \tag{11}$$

where ρ_i, L_i represent the weight density and length of the i truss member

$$A_i = \operatorname*{Max}_{j=1}^{J} \frac{|F_{ij}|}{C_i} \tag{12}$$

$$F_{ij}(R_{kj}) = \sum_{k=1}^{K} \alpha_{ik} R_{kj} + \beta_{ij} \tag{13}$$

and

$$C_i = \begin{cases} \sigma_i^+ \,;\ \text{if } F_{ij} \ge 0 \text{ tension} \\ \sigma_i^- \,;\ \text{if } F_{ij} < 0 \text{ compression} \end{cases} \tag{14}$$

Note that I, J, and K stand for the total number of members, load conditions, and redundants, respectively. Also, Eq. (13) follows from the primary structures equilibrium relations and α_{ik} and β_{ij} are constants for given configurations and load conditions. Upon reflection it will be apparent that when the A_i are evaluated by using Eqs. (12-14) the yield constraints [Eq. (10)] are always satisfied. Therefore, the minimum weight solution which satisfies both the equilibrium and the yield constraints can be obtained by finding the unconstrained minimum of $W(R_{kj})$. Pearson employed a method of random steps, which uses only function evaluations, to find the minimum of $W(R_{kj})$. The choice of algorithm may have been motivated by recognition of the fact that although the $W(R_{kj})$ function is continuous it exhibits local first derivative discontinuities. The particularly fascinating aspect of Pearson's approach is that it simultaneously seeks an optimum design and the critical collapse mechanisms.

Background and Experience Factors

The state-of-the-art in structural optimization as of 1958, my technical background, and the trace of certain particular experiences triggered the thinking which led me to set forth the structural synthesis concept in 1960 (Ref. 17). In this section, I will recall some of the early experiences that influenced the development of my ideas in the structural optimization field.

My earliest interest in structural optimization goes back to 1949-1950, when I wrote a thesis entitled "Optimum Column Stiffening" under the supervision of the renowned and venerable Professor Charles H. Norris at M.I.T. The thesis dealt with the problem of determining a minimum weight reinforcement for existing columns that were to carry higher loads than those for which they were originally designed. Although this work was done without the use of modern digital computers, one measure of progress in structural optimization is that today the optimum column stiffening problem can be given as a homework assignment. Indeed, although the thesis was my first venture in structural optimization, today what I remember most vividly about it is the (not so funny at the time) comment of a rather caustic chemistry major who remarked, upon learning the title, "my word, how phallic."

It was not until 1957 that I really began to work on the structural synthesis idea in earnest. However, my perspective, motivation, and approach were significantly influenced by experience as a stress analyst at the Grumman Aircraft Engineering Corporation (GAEC '51-'53) and then as a structures research engineer in the Aeroelastic and Structures Research Laboratory (ASRL '53-'58) at M.I.T.

As a stress analyst, I took satisfaction in being able to apply analytical methods to predict the behavior of structural systems. On the other hand, I learned that these analytical predictions were often arrived at only after the design had been frozen. Furthermore, because these analyses usually involved considerable simplification, there was a strong tendency to rely heavily on testing. This situation started to change as primitive digital computers became commercially available, around 1951. While prudence demanded that initially the new computing power be used to execute the existing analysis procedures faster and cheaper, it also provided a special opportunity to upgrade the modeling of structural systems. A vast accumulation of numerical analysis procedures, including matrix methods of structural analysis, began to be applied. Prediction of the behavior of trial designs was becoming ever more reliable, and results could even be obtained soon enough to influence the design before it was frozen.

Working with designers as a stress analyst and having them respond to analysis results by saying things like: the stresses in these skins are *too high*; the stresses in these skins are *too low*; the tip deflection of this wing is *too large*; that fundamental frequency is *too low*; was to play a central role in the formation of the structural synthesis concept—after all, this language implies the notion of inequality constraints.

When the results of structural analyses showed the trial design to be inadequate, the redesign procedure was far from clearly defined. Instead, it was an artful combination of judgment experience, and often courage. Digesting the copious output from computer analysis of the current trial design, making the design modification decisions, inputting the modified design so that its behavior could be predicted by the analysis program, all took considerable calendar time. I could not help wondering why more of this redesign could not be done by the computer, eliminating the need for repeatedly crossing the man-machine interface. The problem, it seemed to me, was to find a way to use the computer to do more than just structural analysis of trial designs.

As a research engineer at ASRL from 1953 to 1958, I was involved in research aimed at developing tools for predicting the behavior of aircraft structures subject to nuclear weapons and aerodynamic heating effects. These were challenging assignments and they strengthened my knowledge of analytical and numerical methods in structural mechanics. The Aeroelastic and Structures Research Laboratory in the 1950's was a vibrant and exciting research environment. The stimulation of working with people like Ray Bisplinghoff, Jim Mar, Ted Pian, and Holt Ashley is hard to describe or recapture. However, I vividly remember that the ASRL research environment encouraged lateral and independent thinking—there was never any shortage of new ideas. While doing research at ASRL, I became immersed in transient temperature analysis, finite difference techniques, finite element methods, variational calculus, as well as aeroelastic and nonlinear behavior of structures.

In terms of the yet unborn structural synthesis concept, the special opportunity that ASRL provided in terms of access to a pace-setting digital computing facility was very important. Back in 1954, the Whirlwind I machine was available to ASRL staff between 6 p.m. and 6 a.m.—originally coding was in absolute machine language, later in symbolic assembly language and octal dumps of core provided a debugging tool of sorts. This, after all, was digital computing in the pre-FORTRAN era. Yet by burning the midnight vacuum tubes, it was possible to accomplish a great deal. One of the features of Whirlwind I was that it had cathode ray tube display capability which could be used for graphic as well as alpha numeric output. As a result of using the Whirlwind I machine to solve a variety of complex structural problems, I had a good idea of the computer power available as well as some notion of future expectations. This was to become important when it later became apparent that implementation and demonstration of the structural synthesis concept would require substantial computing capability.

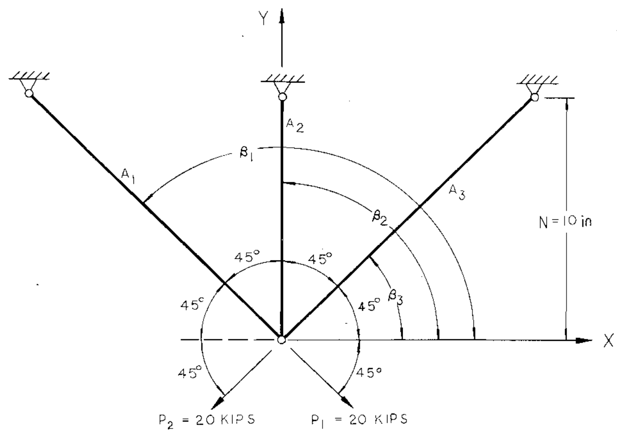

Fig. 9 Three-bar truss example.

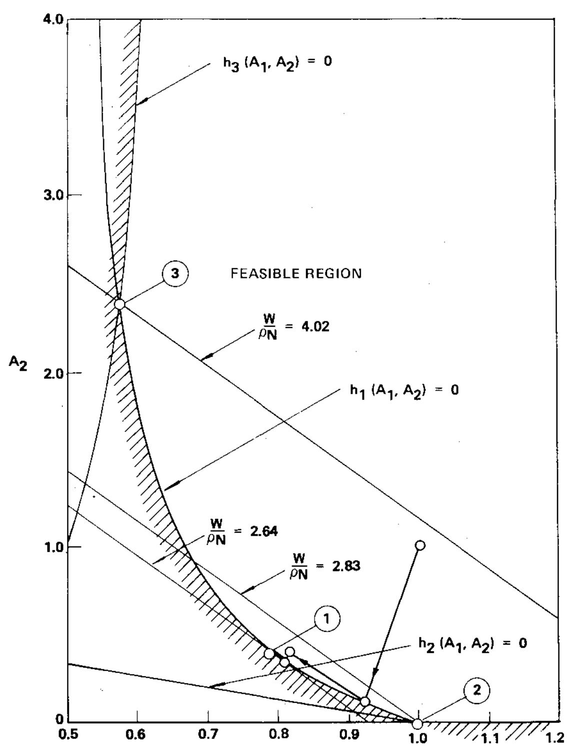

Fig. 10 Design space map for three-bar truss.

The Structural Synthesis Concept Emerges

One day in 1957, I decided to attend a seminar on linear programming applications in electrical networks given by Dean Arden at M.I.T. As I listened, it occurred to me that many of the structural design modification problems that I had been frustrated by at GAEC could be posed as inequality constrained minimization problems. After all, minimum weight structural design could be viewed as an allocation of scarce resources problem and hence it should be possible to bring the mathematical tools of operations research to bear on such problems.

The idea fascinated me, and I decided to explore it using a simple example. In particular, I considered a symmetric three-bar planar truss (see Fig. 9) mapping the design space (Fig. 10), and trying out various primitive solution schemes. These early efforts worked out well, and they stimulated my interest. Ray Bisplinghoff encouraged me to pursue the idea, and he suggested a joint venture with Arthur D. Little, Inc. (ADL) aimed at trying to raise research support from a consortium of aircraft companies. As a result of following this suggestion, I met Carl Pearson, who was then at ADL. Although we were unsuccessful in putting together a consortium, Pearson and I had a fruitful exchange of ideas. When, in the summer of 1958, I left ASRL to go teach at Case Institute of Technology, we agreed that Pearson would pursue the application of mathematical programming methods to structural optimization in the context of the plastic collapse design philosophy, while I would work on their application within the framework of the elastic service load design philosophy. This was a rather natural division, since Pearson was more interested in civil engineering structures, while I was primarily interested in aerospace structures. As I left ASRL, Ray Bisplinghoff generously encouraged me to take my ideas and early work on applying operations research methodology to structural design with me. As a young assistant professor at Case, I vigorously pursued my structural optimization ideas as well as research funding for their further development.

By 1960, I was ready to present a rather general new approach to structural optimization which I called structural synthesis, as well as some numerical results for simple truss examples obtained on an IBM 653 machine. In Ref. 17 I set forth a rather general philosophy which began with the following definition: "Systematic structural synthesis may be defined as the rational directed evolution of a structural configuration which, in terms of a defined criterion efficiently performs a set of specified functional purposes." The paper went on to point out that "the structural design problem could be looked at as a problem in the programming of interdependent activities" and that "three types of considerations were involved in such problems: (I) a specified set of requirements, (II) a given technology and (III) a criterion by means of which choices can be made between various solutions." The analogous phases of the structural design problem were identified as follows:

I. Specifications and requirements: 1) establish a set of multiple distinct loading conditions, including mechanical and thermal loads; 2) define a set of failure modes to be guarded against, such as limitations on stress levels and deflection magnitudes; and 3) assign upper and lower limits on member sizes in order to preclude impractical designs.

II. Technology: An appropriate method of analysis for predicting behavior relative to the failure modes guarded against constitutes a component part of structural synthesis. In many cases, "lumped element structural idealizations followed by matrix formulation of the structural analysis problem will be suitable."

III. Criterion: In many structural design areas the minimization of weight is important. It is, however, theoretically possible to employ other criteria, such as cost minimization, if enough is known to construct a meaningful cost objective function.

The important, unique contribution of Ref. 17 was that it introduced the idea and indicated the feasibility of coupling finite element structural analysis and nonlinear mathematical programming to create automated optimum design capabilities for a rather broad class of structural systems. Working within the elastic design philosophy, I showed that the minimum weight optimum design of elastic statically indeterminate structures could be posed as a nonlinear mathematical programming problem in design variable space. The general formulation set forth in Ref. 17 emphasized the importance of considering a multiplicity of distinct loading conditions and guarding against a variety of failure modes by using inequality constraints. The inclusion of minimum and maximum member size constraints was another practical feature that was not ignored. Because the design optimization problems treated in Ref. 17 had the form of a nonlinear mathematical programming problem, it followed that the optimum design did not necessarily lie at a vertex in the design space. Therefore, it was pointed out in Ref. 17 that, contrary to the commonly held viewpoint, the minimum weight design for a statically intermediate structure, subject to stress constraints only, is not necessarily one in which each member is fully stressed in at least one load condition.

The fact that some of the three-bar truss results reported were counterintuitive, in the sense that they were *not fully stressed* designs,† heightened interest in Ref. 17. It also served to bring attention to the potential flaw in the basic premise of the simultaneous failure mode method, which was the prevailing approach to aircraft structural component optimization at the time (1960). In view of the impact these three-bar truss examples had, the best known example is briefly summarized here.

Consider the symmetric three-bar planar truss shown in Fig. 9. Two distinct load conditions are to be considered, the first specified by $P_1 = 20{,}000$ lb acting at an angle of 45 deg to the x axis and the second specified by $P_2 = 20{,}000$ lb acting at an angle of 135 deg to the axis. The configuration descriptors ($N = 10$ in., $\beta_1 = 135$ deg, $\beta_2 = 90$ deg, $\beta_3 = 45$ deg) and the material properties ($\rho = 0.1$ lb/in.^{3}, $E = 10\times 10^6$ lb/in.^{2}) are preassigned parameters. Because the truss is to be symmetric, $A_1 = A_3$ and, therefore, the two independent design variables are A_1 and A_2. The problem posed is to find A_1 and A_2 such that the total weight is minimized subject to the requirements that the member stresses lie between a compressive allowable of $-15{,}000$ lb/in.^{2} and a tensile allowable of 20,000 lb/in.^{2}; that is: Find A_1 and A_2 such that

$$W(A_1,A_2) = \rho N(2\sqrt{2}A_1 + A_2) \rightarrow \text{Min} \qquad (15)$$

and

$$-15{,}000 \leq \sigma_{ij}(A_1,A_2) \leq +20{,}000: \quad i=1,2,3; \quad j=1,2 \qquad (16)$$

where it is understood that σ_{ij} denotes the stress in the ith member under the jth load condition. Elementary indeterminate structural analysis gives expressions for the σ_{ij} in terms of the design variables A_1 and A_2. Because of symmetry, only three of the six inequality constraints embodied in Eq. (16) need be considered, and they have the algebraic form

$$h_1(A_1,A_2) = 20{,}000 - \sigma_{11} \geq 0 \qquad (17)$$

$$h_2(A_1,A_2) = 20{,}000 - \sigma_{21} \geq 0 \qquad (18)$$

$$h_3(A_1,A_2) = 15{,}000 + \sigma_{31} \geq 0 \qquad (19)$$

where

$$\sigma_{11} = 20{,}000\left[\frac{1}{A_1} - \frac{A_2}{2A_1A_2 + \sqrt{2}A_1^2}\right] \qquad (20)$$

†A fully stressed truss design is understood to be a truss in which every member is stressed to its allowable limit in at least one load condition.

$$\sigma_{21} = \frac{20{,}000\sqrt{2}A_1}{2A_1A_2 + \sqrt{2}A_1^2} \quad (21)$$

$$\sigma_{31} = -\frac{20{,}000A_2}{2A_1A_2 + \sqrt{2}A_1^2} \quad (22)$$

The significant portion of the design space for this example is shown in Fig. 10. Scanning the design space, we see that the minimum weight optimum design lies at point ① (i.e., $A_1 = A_3 = 0.788$ in.^{2}, $A_2 = 0.41$ in.^{2}, and $W = 2.64$ lb). This optimum design does not lie at a vertex, and it represents an indeterminate structure in which member 2 is not fully stressed in either load condition. The design represented by point ② in Fig. 10 (i.e., $A_1 = A_3 = 1.0$ in.^{2}; $A_2 = 0$ in.^{2} and $W = 2.83$ lb) is not the minimum weight optimum design even though it is a) at a vertex, b) determinate, and c) fully stressed. The design represented by point ③ in Fig. 10 is a) at a vertex, b) indeterminate, and c) not fully stressed. This example illustrated by counter example that fully stressed designs (FSD), which amount to a particular kind of simultaneous failure mode design, are not necessarily minimum weight optimum designs.

So far, all of the examples considered have involved only two independent design variables, and therefore it has been possible to map the constraint and objective function contours. Two variable design space plots are helpful for introducing the basic concepts of structural synthesis while initially avoiding the mathematical abstraction and associated generality. However, it must be clearly recognized that when the mind's eye scans a two-dimensional design space (e.g., Figs. 2, 4, 5, and 10) and locates the optimum design, the method employed amounts to a brute force "try them all" approach. In practice, design modification schemes provide rational methods for iterative design improvement, and the efficiency of such design procedures is often measured by how few designs we have to try before converging to an optimum design.

The algorithm used to generate numerical solutions for the several three-bar truss examples reported in Ref. 17 was a primitive feasible directions method called the "method of alternate steps." The alternate step design modification trajectory is shown schematically in Fig. 10 for the particular three-bar truss problem posed by Eqs. (15) and (16). For three-bar truss problems with three independent design variables and two or three distinct load conditions, Ref. 17 reported that less than 30 min of IBM 653 time was required to converge to a design having a weight within 1% of the minimum weight design. In retrospect, I am inclined to observe that given such run times only a congenital optimist could have been so enthusiastic about future prospects. It is apparent upon review of my early structural synthesis work that I was not inhibited by a great deal of formal mathematical programming knowledge. Oddly enough, that may help to account for my boldness in pursuing the structural synthesis concept with the intense enthusiasm of a "true believer."

In any event, as of 1960, it was known, if not widely appreciated, that a rather general class of structural design optimization problems could be stated in standard form as follows: given the *preassigned parameters* and *load conditions* find the vector of design variables $\boldsymbol{D}$ such that

$$g_q(\boldsymbol{D}) \geq 0; \quad q = 1,2,\ldots Q \quad (23)$$

and the *objective function*

$$M(\boldsymbol{D}) \rightarrow \text{Min} \quad (24)$$

where the set of inequality constraints represented by Eq. (23) usually contains one behavior constraint for each failure mode in each load condition as well as side constraints that reflect fabrication and analysis validity considerations. Although the validity of this problem statement has not been seriously challenged, the last two decades have seen a great deal of controversy over how to solve it efficiently for practical structures.

The First Structural Synthesis Research Grant

In the summer of 1958, I arrived in Cleveland to take up my responsibilities as an Assistant Professor at Case Institute of Technology. Under the leadership of Keith Glennan and John Hrones, the Case School of Applied Science, which had been primarily an undergraduate school, was being transformed into an Institute of Technology with an active graduate level educational program involving research. This was part of a national trend in the immediate post-Sputnik era. My Department Head, Harry Nara, who had brought me to Case, was a very strong supporter of my research ideas. In addition, he provided tangible support in the form of reduced teaching loads during my first two years at Case. I used the time that Nara's commitment gave me to vigorously pursue the structural synthesis concept and to seek financial support for its further development.

By April of 1959, I had generated a research proposal that included most of the basic ideas subsequently set forth in Ref. 17. My experience at ASRL before coming to Case had included participation in the preparation and presentation of several successful research proposals. During 1959 and 1960 I learned that obtaining support for research under the leadership of an established principal investigator at a premiere institute of technology in Massachusetts was a lot easier than finding sponsorship for the research ideas of a young assistant professor at a budding graduate institution in Ohio. During the two years of trying to find a research sponsor, I wrote to and sought appointments with most of the agencies that have traditionally sponsored research in structural mechanics. In addition to "no," there were two other common responses to my first proposal for development of the structural synthesis concept. The first was that Shanley and Gerard had already done it, and the second was that what I proposed was interesting but hopelessly complicated and impractical.

The turning point in my quest for a research sponsor came in June 1960, when I met Mel Rosche at NASA Headquarters for the first time. He had read my proposal which was then pending in the Office of University Affairs. During this meeting, I explained my ideas with enthusiasm as well as conviction and somehow I added new clarity that was missing in the written document. It turned out that the structural synthesis concept had struck a resonant chord in Rosche's lively imagination. On his initiative, I was given a full hearing at the Langley Research Center on August 3, 1960. In the morning, I presented the general structural synthesis concept and explained how it differed from the existing structural optimization methods. After lunch, I showed some of the example problems for which solutions had been obtained on the IBM 653 at Case. As I remember it, this meeting was more like an extended thesis defense than anything else, and the examining committee included Paul Kuhn, Roger Anderson, Dick Heldenfels, John Hedgepeth, and Mel Rosche.

Some of the key features of the structural synthesis approach that helped carry the day were that it could: 1) include lower and upper limits on member sizes as well as other simple rules of thumb (e.g., limits on D/T ratios, etc.); 2) take into account a multiplicity of distinct loading conditions, including mechanical and thermal loads; 3) guard against a wide range of failure modes including stress, displacement, buckling, etc.; and 4) accommodate objective functions other than weight minimization.

However, it was made very clear that three-bar trusses were not very interesting to NASA! It was John Hedgepeth who asked "can't you treat something that looks more like part of an airplane or a spacecraft, for instance an integrally stiffened

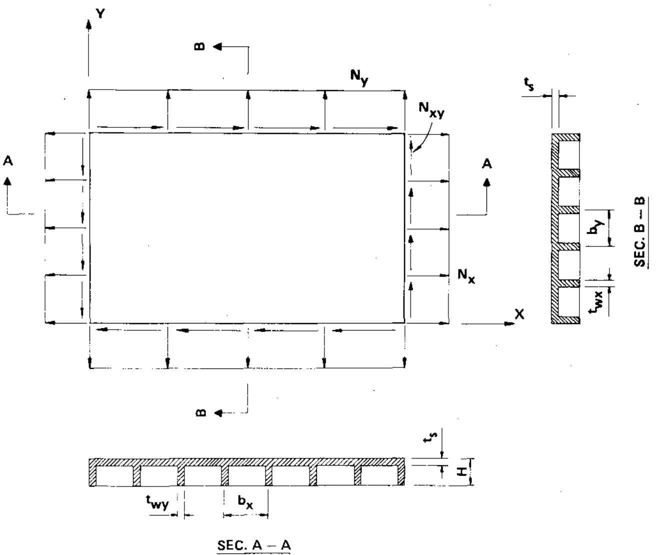

Fig. 11 Integrally stiffened waffle plate.

waffle plate panel?" (see Fig. 11). My response on the spot was somewhat guarded, although I could see that going beyond the three-bar truss was now crucially important. In effect I said yes, provided we can agree upon an appropriate structural analysis for the load conditions and failure modes to be considered. The conclusions reached at the August 3, 1960 meeting at Langley Research Center were:

1) Research in the general area of structural synthesis should be pursued.

2) Effort should be directed toward developing structural synthesis capabilities for aerospace structures of current and future importance (e.g., integrally stiffened waffle plates).

It was now clear to me that funding was at hand and I returned to Case where Tom Kicher (my first graduate student) and I formulated the integrally stiffened waffle plate problem as a nonlinear programming problem and took a preliminary cut at the buckling analyses that would be required. Within three weeks, it was sufficiently clear that an appropriate structural analysis (on which to base the waffle plate synthesis) could be generated, and on August 25, 1960, I submitted a revised letter-type proposal to Mel Rosche at NASA Headquarters. The first structural synthesis research grant was awarded to Case Institute of Technology by NASA, effective December 1, 1960.

By March of 1961, Tom Kicher and I had prepared a report detailing the structural analysis on which the waffle plate synthesis would be based. Each load condition was specified by assigning the inplane force resultants N_x, N_y, and N_{xy} (see Fig. 11) and the failure modes considered were: overall buckling of the waffle plate, local instability of the stiffeners, local instability of the sheet, and material yield in the sheet and stiffeners. By November 1961, Kicher and I were able to report[18] successful development of a structural synthesis capability for symmetric waffle plates of preassigned total depth [i.e., the three independent design variables were t_s, $t_{wx}=t_{wy}$, and $b_x=b_y$ (see Fig. 11)]. This work was well received at the Langley Research Center and, following some revisions to incorporate more conservative boundary conditions in the local buckling analyses, we were asked to prepare a summary report in the format of a NASA Technical Note.[19]

It was in the course of this research that Kicher encountered relative minima in the design space. At first, my reaction to this was disbelief, because I had a strong intuitive bias toward the notion that a structural optimization problem should have a unique solution. However, Kicher persisted and careful study of the design space mappings confirmed that relative minima were indeed present. I still remember the three-dimensional plexiglass model that Kicher made to illustrate the existence of relative minima in the symmetric waffle plate design space. These relative minima were due to the fact that the overall panel buckling constraint functions were not convex. In a physical sense, they could be interpreted as subconcept pockets within the design space (i.e., thick sheet and stiffened thin sheet pockets). In September of 1961, when Tom Kicher and I went to NASA to report on our progress, we drove to Langley and took along the three-dimensional design space model in the trunk. However, we realized that this type of visual aid would have to be abandoned as we moved forward to design problems with more than three independent design variables.

Meanwhile, Bill Morrow (my second graduate student) had started working on the general unsymmetric waffle plate problem, which involved seven independent design variables (see Fig. 11). Our efforts to generate a structural synthesis capability for integrally stiffened waffle plates culminated in the presentation of Ref. 20 at the AIAA Launch and Space Vehicle Structures Conference at Palms Springs, California in April 1963. This paper was well received, and I remember George Gerard offering his kind remarks from the floor following the presentation. It was particularly gratifying to hear him say that he agreed with our finding that minimum weight optimum designs did not always correspond to a simultaneous failure mode vertex in design space. The firm conclusions drawn in Ref. 20 were:

1) The development of a structural synthesis capability for waffle plates represents a considerable advance beyond the exploratory truss studies and adds to the growing body of evidence supporting the contention that a structural synthesis capability can be developed for complex structural systems of practical importance.

2) Future structural synthesis research should seek to bring an ever more meaningful class of structural design problems within the grasp of automated optimization.

This was, in a sense, the end of the beginning of the structural synthesis concept.

Sequel

During the decade 1960-1970, immediately following the introduction of the structural synthesis concept, progress was made along two main lines. In the first category the concept was applied to component type problems of fundamental and recurring nature, for example, see Refs. 21-26. The second broad category of activity involved the development of first generation system level structural synthesis programs based on joining finite element analysis and nonlinear mathematical programming algorithms, for example, see Refs. 27-31.

The component type problems were characterized by: 1) relatively small numbers of design variables; 2) a wide variety of increasingly complex failure modes and loading environments; and 3) in some instances consideration of objective functions other than weight (e.g., Refs. 21 and 22). These component type problems included automated optimum design of: a simple shock isolation system (Ref. 21); a highly idealized double wedge wing subject to a mix of constraints limiting flutter Mach number, static aeroelastic displacements, combined stress, and angle of attack (Ref. 22); an ablating thermostructural panel subject to time dependent heat flux and dynamic pressure inputs (Ref. 23); stiffened cylindrical shells considering buckling, strength and minimum gage constraints (Refs. 24-26).

The first application of mathematical programming methods to the minimum weight optimum design of stiffened shell structures was made by Kicher.[24] Subsequently a structural synthesis capability for minimum weight design of stiffened cylindrical shells (see Fig. 12) representative of the state-of-the-art (circa 1968) was presented in Ref. 25. It is important to elaborate briefly on Ref. 25 because of the influence it was to have on future developments. The mathematical programming problem statement of the structural design optimization task [see Eqs. (23) and (24)]

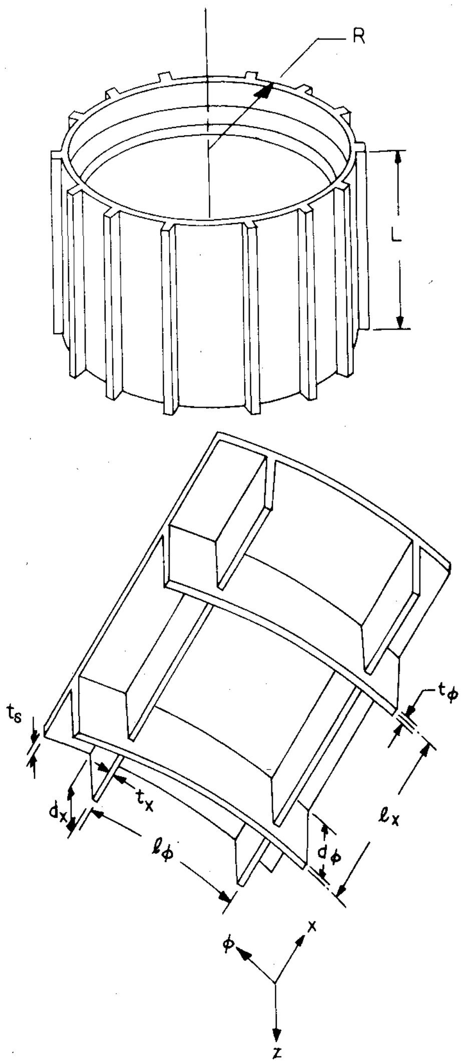

Fig. 12 Stiffened cylindrical shell.

was transformed into a sequence of unconstrained minimizations using the Fiacco-McCormick interior penalty function formulation; that is, find $\boldsymbol{D}$ such that

$$\phi(\boldsymbol{D},r_0) \rightarrow \text{Min} \tag{25}$$

where

$$\phi(\boldsymbol{D},r_p) = M(\boldsymbol{D}) + r_p \sum_{q=1}^{Q} \frac{1}{g_q(\boldsymbol{D})} \tag{26}$$

$$r_{p+1} \leftarrow Cr_p; \quad C<1 \tag{27}$$

The constraint repulsion characteristic of this interior penalty function formulation causes successive designs obtained during the synthesis to stay away from the constraints. This led to the idea that approximate analyses could be used during major portions of the synthesis with a good expectation that the design generated would remain in the feasible region of the design space. Indeed, by using approximate buckling analyses within each unconstrained minimization stage, analysis run times were reduced by a factor of about 75, while still generating a sequence of steadily lighter and noncritical feasible designs which form a trajectory that tends to "funnel down the middle" of the feasible region in design space. In a philosophical sense, this approximate analysis feature was a precursor of the approximation concepts approach to structural synthesis which was to emerge during the next decade (1970-1980).

An extension of the capability contained in Ref. 25 to the minimum weight design of barrel shells was reported by Stroud and Sykes in Ref. 26. The following quotation from

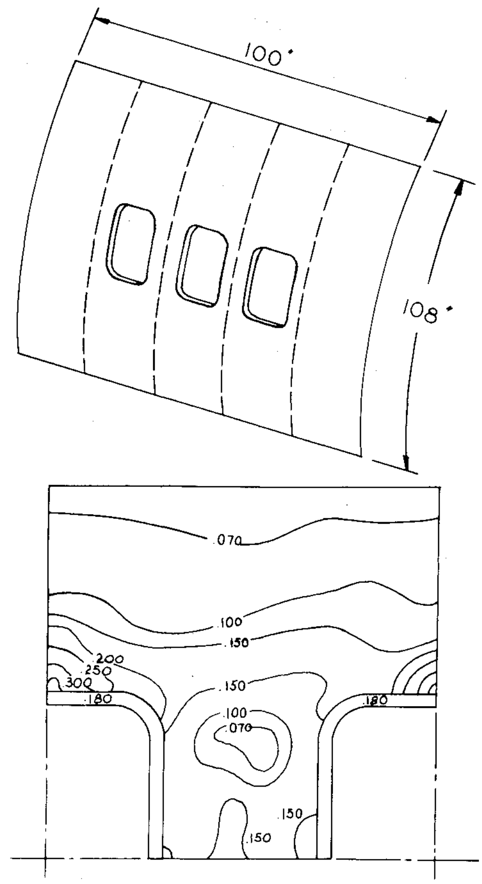

Fig. 13 Window panel application.[31]

Ref. 26 serves to illustrate the important role structural synthesis capabilities can play in evaluating new design concepts:

> For shells designed to support axial compressive loads, the results show that important weight savings can be provided by slight meridional curvature. For the particular shell examined herein, the maximum weight saving is about 30%. The large increases (factors of 5 to 9 in strength) recently attributed to barreling cannot be directly translated into weight savings when comparisons are made between minimum weight designs. Yielding becomes an important failure constraint at lower loads for barrel shells than for cylindrical shells.

During the 1960-1970 time frame two system level structural synthesis programs were developed by combining finite element and mathematical programming algorithms. The first major efforts along this line were led by Gallagher and Gellatly at Bell Aerosystems (see Refs. 27-29). Subsequently, a second major effort in this category was carried out by Tocher and Karnes at Boeing (see Refs. 30 and 31). In this work special emphasis was placed on least weight design of stressed skin structures with holes and cutouts (see Fig. 13). The capability reported in Ref. 31 represented the most general and sophisticated structural synthesis program available at the end of the 1960-1970 decade. The structural analyses were executed using an efficient finite element displacement method module and the optimization was accomplished by a sound implementation of Zoutendijk's feasible directions algorithm. A form of design variable linking was included so that the number of design variables was independent of the number of finite elements employed in the structural analysis model. The importance of reducing the number of structural analyses and the number of partial derivative calculations was recognized and several devices aimed at improving overall efficiency of the design optimization procedure were introduced. Various techniques were employed to improve the efficiency of the one-dimensional searches and partial derivatives were only recalculated when the design moved outside of a user defined hypersphere. The largest problem reported in Ref. 31 (an application to the design of the honeycomb face sheets for a 747 aircraft window panel test specimen—see Fig. 13) involved 600 finite elements, 300 nodes, and 5 load conditions. While run times were not reported in Ref. 31, it is known that they were substantial (in excess of 1 hour on a CDC 6600).

The rapid development of the structural synthesis concept during the 1960's stimulated a great deal of interest in structural engineering applications of mathematical programming techniques. By 1970 enough had been accomplished to warrant a brief period of consolidation and assessment characterized by review papers and educational endeavors. For example, Ref. 32 traces progress reported during the 1960's and Ref. 33 presents a comprehensive and detailed description of the state of the structural synthesis field as of 1970. Growing interest in the future potential of the field also led to educational endeavors such as the AIAA Professional Studies short course on structural synthesis given in April 1970[34] and the publication by Fox in 1971 of an excellent text book on algorithmic tools for engineering design optimization.[35]

Nevertheless, by 1970 it had become apparent that the available optimization capabilities based on combining finite element analysis with mathematical programming techniques required inordinately long run times to solve structural design problems of only modest practical size. Indeed, in 1971 (see Ref. 36) the decade 1960-1970 was characterized as a "period of triumph and tragedy for the technology of structural optimization," and it was suggested that the mathematical programming approach to structural optimization was little more than "an interesting research toy." Furthermore, it was also stated in Ref. 36 that "there appeared little immediate prospect for the development of more efficient nonlinear programming algorithms to overcome the economic barriers to widespread operational usage on real structures." This grim assessment of the structural synthesis concepts future had two effects:

1) It led one group of investigators to expend renewed effort on the implementation of redesign procedures based on fully stressed design concepts and discretized optimality criteria.

2) It led other investigators to focus effort on improving efficiency, while preserving the philosophically attractive generality of the mathematical programming approach.

Fully stressed design (FSD) methods are based on simple stress ratio type recursive redesign rules for modifying member sizes at each iteration so as to make the maximum stress in each member equal to the allowable stress (in at least one load condition) while assuming no force redistribution. The underlying and potentially flawed premise of these methods is that they presume a fully stressed design is equivalent to a minimum weight design. Fully stressed designs correspond to vertex points in design space, and they are a special kind of simultaneous failure mode design.

Optimality criteria methods for structural optimization involve first the derivation of a set of necessary conditions that must be satisfied at the optimum design and then the development of an iterative redesign procedure that drives the initial trial design toward a design which satisfies the established set of necessary conditions. Design procedures based on optimality criteria methods usually involved two distinct types of approximations: those associated with identifying how many and which constraints will be critical at the optimum design; and those associated with the development of iterative redesign rules. During the late 1960's and early 1970's, many investigators focused their efforts on constructing automated structural design procedures based on fully stressed design and discretized optimality criteria concepts (e.g., Refs. 37-46). These efforts to create practical automated design tools for large scale structural systems culminated in the development of the ASOP and FASTOP computer programs.[41,47-53]

The early 1970's also saw the beginning of research activity aimed at automated interdisciplinary aerospace vehicle design (see, for example, Refs. 54-60). A thoughtful review and assessment of this activity as of 1973 will be found in Ref. 60. The structural optimization modules in these large scale capabilities were based on the combined use of fully stressed design methods and mathematical programming methods (see Refs. 54 and 57). The general approach followed in the early 1970's is well characterized by the mixed method described in Ref. 57, where a fully stressed design method was used to obtain a gross overall distribution of material while the detailed design of rings and stiffened panels (fuselage components) was carried out using mathematical programming techniques. This approach reflected the widely held view during the early 1970's that although mathematical programming methods were viable for component optimization, they were computationally impractical for dealing with large structural systems of practical importance.

The continuing active development of design procedures for finite element structural systems stimulated interest in design oriented structural analysis methods (see Refs. 61-69). These papers reflected a growing realization that analysis for design optimization is a task with special characteristics. It was recognized that design optimization required behavior prediction for many structures of somewhat similar form.[61] Also, new attention was given to the idea that, in the design context, the objective of structural analysis should be to generate, with minimum effort, an estimate of the critical and potentially critical response quantities adequate to guide the design modification process. Developments in design oriented structural analysis fall into three categories: 1) methods for obtaining rates of change of response quantities with respect to design variables, i.e., sensitivity analysis (e.g., Refs. 62 and 63); 2) techniques for constructing approximate analysis solutions using a few well chosen basis vectors (e.g., Refs. 61, 65, 66), these methods are analogous to the common practice in dynamic analysis of using a reduced set of generalized coordinates and normal mode basis vectors; and 3) reexamination of how finite element methods are organized, focusing on how to improve their organization so that they lend themselves better to the design optimization task (e.g., Ref. 69).

The main obstacles to the implementation of efficient mathematical programming based structural synthesis methods prior to 1970 were associated with the fact that the general formulation of the basic structural design problem involves: 1) large numbers of design variables; 2) large numbers of inequality constraints; 3) many inequality constraints that are computationally burdensome *implicit* functions of the design variables. The introduction of approximation concepts[70] leading to a sequence of tractable approximate problems via the coordinated use of design variable linking (and/or basis reduction), temporary constraint deletion (regionalization and truncation), and the construction of high quality *explicit* approximations for retained constraints (using intermediate variables and Taylor series expansions), has led to the emergence of mathematical programming based structural synthesis methods that are computationally efficient (e.g., Refs. 71-76).

During the late 1960's and early 1970's the application of design procedures based on stress ratio and optimality criteria methods to large finite element systems was a necessary expedient because of the absence of computationally efficient alternatives. However, it was recognized that design procedures based on optimality criteria and fully stressed design concepts could only be shown to yield optimum designs under rather restrictive special conditions. As first noted in Ref. 77, the essential difficulties involved in applying optimality criteria methods to the general structural synthesis problem are those related to identifying the correct critical constraint set and the proper corresponding set of passive members (see also Refs. 44 and 46). These difficulties were recognized and addressed with varying degrees of success in studies such as those reported in Refs. 78-80. However, it was only with the advent of the dual formulation set forth by Fleury in Refs. 81 and 82 that these obstacles were overcome conclusively. Introduction of the dual formulation resolves the essential difficulties inherent to the optimality criteria because determining the critical constraint set and keeping track of the status of each design variable (active or passive) becomes an intrinsic part of the mathematical programming algorithm used to find the maximum of the dual function subject to nonnegativity constraints. In Ref. 83, the dual formulation was interpreted as a generalized optimality criteria method, and it was shown to be well suited to the efficient solution of structural design optimization problems with relatively few critical constraints. In Refs. 84 and 85, the dual method was presented as a basis for coalescing of the mathematical programming and optimality criteria approaches to structural synthesis.

During the past three years, approximation concepts have been combined with the dual formulation to create a powerful new method for minimum weight design of structural systems.[86-88] The method has also been successfully extended to deal with pure discrete and mixed continuous-discrete variable problems. Approximation concepts are used to convert the general structural synthesis problem into a sequence of *explicit* primal problems of separable algebraic form. The dual method formulation, which exploits the separable form of each approximate problem, is used to construct a sequence of explicit dual functions. These dual functions are maximized subject to nonnegativity constraints on the dual variables. The efficiency of the method is due to the fact that the dimensionality of the dual space, where most the optimization effort is expended, is relatively low for many structural optimization problems of practical interest. Furthermore, in the dual formulation, the only inequality constraints are simple nonnegativity requirements on the dual variables.

The combination of approximation concepts and dual methods provides a firm foundation for the further development of rather general and highly efficient structural synthesis capabilities. Although ACCESS 3 is a research type program, the results reported in Refs. 86-88 demonstrate that combining approximation concepts and dual methods leads to a powerful capability for minimum mass optimum sizing of structural systems subject to stress, deflection, slope, minimum gage, and natural frequency constraints. For the class of sizing problems treated in Refs. 86-88 the approximation concepts approach generates explicit constraints that are identical to those employed in the optimality criteria techniques (Refs. 84 and 85). Thus, in a sense, the joining together of approximation concepts and dual methods has led to the envelopment of the optimality criteria method within the general framework of the mathematical programming approach to structural optimization. As Fleury shows in Ref. 89, "the mathematical programming and the optimality criteria approaches, far from being ineluctably opposed, have in fact converged toward the same method."

Structural synthesis is finding increased application in the aerospace and the automotive[90,91] structures community at both the component[92-94] and the system level.[95-98] It is also interesting to note that many of the ideas and mathematic programming methods successfully employed in the structural synthesis context have subsequently found application across a wider range of engineering design problems (see Ref. 99), including conceptual vehicle synthesis (e.g., Ref. 100) and optimum design of airfoils (e.g., Ref. 101).

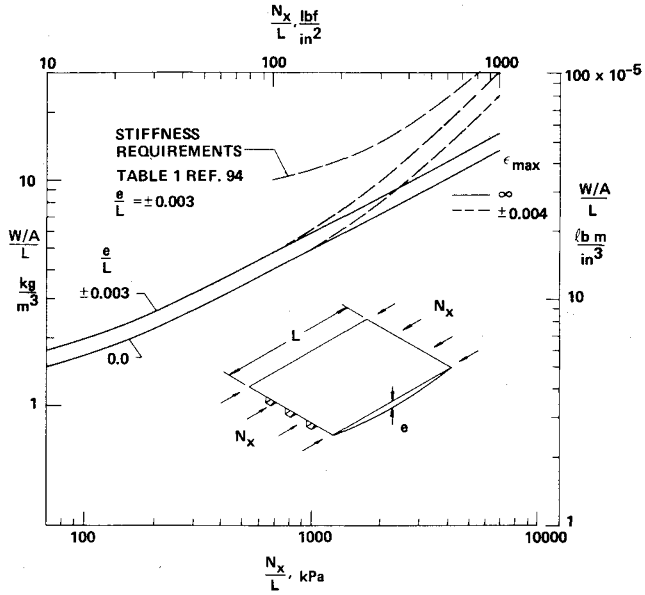

Fig. 14 Structural efficiency of graphite-epoxy hat-stiffened panels.[94]

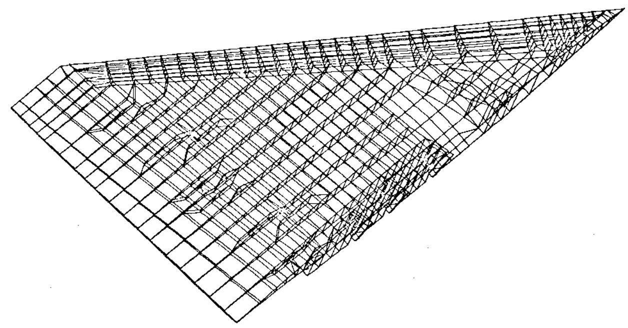

Fig. 15 Idealization for delta wing application.[98]

The panel analysis and sizing code (PASCO) reported in Ref. 94 represents a current state-of-the-art component level structural synthesis capability. Approximation concepts are employed to join a high quality analysis module with an efficient optimizer. The result is a computer program (PASCO) which has sufficient generality (in terms of panel configuration, loading conditions, and practical constraints) so that it can be used for sizing of stiffened panels in a realistic design environment. Figure 14 shows a representative plot of weight index vs loading index drawn from Ref. 94. I find it particularly gratifying to see minimum weight optimum design results obtained via modern structural synthesis methods (in contrast to the old simultaneous failure mode method) being summarized in this time honored and useful manner.

The work reported in Ref. 98 represents a current state-of-the-art system level application of modern structural synthesis methods. The capability described in Ref. 98 includes the use of approximation concepts to generate a sequence of explicit approximate problems as well as the application of sophisticated mathematical programming algorithms. It is reported that convergence is obtained after only three or four stages, and the overall cost of an optimization runs from 8 to 12 times the cost of a simple analysis. Figure 15, from Ref. 98, shows the idealization of a delta wing problem involving 4269 degrees-of-freedom, 88 independent design variables, and 190 static and aeroelastic constraints. Based on Ref. 98, it would appear that structural synthesis has become a routine part of the design process at Avion Marcel Dassault Breguet Aviation.

Since its birth in 1960, structural synthesis has grown from an abstract concept to a practical tool which is currently serving in the quest for better structural designs. Although much remains to be done and it is inevitable that synthesis must lag analysis development, structural design procedures created by combining finite element analysis and mathematical programming algorithms can no longer be dismissed as "research toys." Although structural synthesis has not yet achieved the near universal acceptance level enjoyed by finite element analysis methods, a firm knowledge and experience base exists for the further development of rather general and highly efficient structural synthesis capabilities. As in the case of finite element analysis, usage and acceptance of structural synthesis methodology will grow with the development and distribution of easy to use, well documented production quality computer programs.

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